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ELASTIC EQUILIBRIUM OF ANISOTROPIC SHELLS REINFORCED BY STIFFENER RIBS

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A fundamental solution of the theory of shallow anisotropic shells is constructed, the principal part is extracted and some of its properties are studied. A procedure is indicated for constructing the Green's function for a finite shell. The solution constructed is used in investigating the state of stress of an anisotropic shell in the neighborhood of the point of application of a concentrated force. A solution is given for the problem of elastic equilibrium of an anisotropic shell rein-

forced by a periodic system of finite stiffeners. The problem is reduced to a singular integro-differential equation in the jump in the tangential stress on the line of contact. The results of numerical computations are presented.

The effects of forces concentrated at points and lines on an elastic shell have been studied by many authors (see the reviews [1 – 3]). Asymptotic formulas for the forces and moments in an isotropic shell loaded along the lines of principal curvature have been obtaind in [4, 5]. Transfer of the forces from the thin rib to the cylindrical isotropic shell was first studied in a rigorous formulation in [6].

1. Partucular solutions in the theory of anisotropic shells which correspond to concentrated loads. Let us start from the equilibrium differential equations in displacements for shallow anisotropic shells comprised of an odd number of homogeneous anisotropic layers [7]

$$\sum_{k=1}^{3} L_{kj} u_k = p_j, \quad k, j = 1, 2, 3 \tag{1.1}$$

Here u_j, p_j are components of the displacement vector and of the external load, and L_{kj} are known operators [7].

The operator $B^* = \det\,[L_{kj}]$ is elliptic, of the form

$$B^* = fL\left(\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}\right) \tag{1.2}$$

$$f = \frac{D_{11}A_{22}}{R_2{}^8}\left[(C_{11}C_{22} - C_{12}{}^2)C_{66} + 2C_{12}C_{16}C_{26} - C_{11}C_{26}{}^2 - C_{22}C_{16}{}^2\right]$$

$$L\left(\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}\right) = L_0\left(\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}\right) + a_9\left[\frac{\partial^4}{\partial\alpha^4} + 2\lambda\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + \lambda^2\frac{\partial^4}{\partial\beta^4}\right]$$

$$L_0\left(\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}\right) = \sum_{j=0}^{8} a_j \frac{\partial^8}{\partial\alpha^{8-j}\partial\beta^j}, \quad \lambda = \frac{R_2}{R_1}$$

Here C_{jk}, D_{jk}, A_{jk} are associated with the elastic anisotropy parameters, α and β are dimensionless Cartesian coordinates, R_1 and R_2 are the principal radii of surface curvature. In (1.2), we set the coefficients of the first quadratic form of the surface A and B which are in (1.1), equal to R_2.

It follows from (1.1) and (1.2) that the problem of the effect of a concentrated force is reduced to determining the fundamental solution of the operator L.

Let us construct a fundamental solution E of the operator L which is T-periodic in β. We have

$$L\left(\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}\right)E = \delta(\alpha)\,\delta_T(\beta) \tag{1.3}$$

By virtue of the relationship

$$\delta_T(\beta) = \sum_{k=-\infty}^{\infty} \delta(\beta - kT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{ik\omega\beta} \quad \left(\omega = \frac{2\pi}{T}\right)$$

we seek the functions $E(\alpha, \beta)$ in the form

$$E(\alpha, \beta) = \frac{1}{T}\sum_{k=-\infty}^{\infty} c_k(\alpha)\, e^{ik\omega\beta} \tag{1.4}$$

Substituting (1.4) into (1.3) and equating coefficients of identical powers of $e^{ik\omega\beta}$ in the equation obtained, we are led to an infinite system of linear differential equations to determine $c_k(\alpha)$.

Let us seek $c_k(\alpha)$ in the class D' of generalized functions of slow growth [8]. Using the property of convolution of generalized functions, we can write

$$\left\{\sum_{j=0}^{8} a_j (ik\omega)^j \delta^{(8-j)} + a_9 [\delta^{(4)} - 2\lambda (k\omega)^2 \delta^{(2)} + \lambda^2 (k\omega)^4 \delta]\right\} * c_k = \delta, \quad (1.5)$$
$$k = 0, \pm 1, \pm 2, \ldots$$

Finding the inverse Fourier transform of both sides of (1.5), we obtain

$$F^{-1}(c_k, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} c_k(\alpha) e^{-i\alpha\mu} d\alpha = [2\pi \Pi_k(\mu)]^{-1} \quad (1.6)$$

$$\Pi_k(\mu) = \sum_{j=0}^{8} a_j (k\omega)^j \mu^{8-j} + a_9 [\mu^4 + 2\lambda (k\omega)^2 \mu^2 + \lambda^2 (k\omega)^4] =$$
$$\prod_{\nu=1}^{4} (\mu - \mu_\nu^{(k)}) (\mu - \overline{\mu_\nu^{(k)}})$$

The Fourier transform of the function $[\Pi_k(\mu)]^{-1}$ is known if the imaginary parts of the roots do not vanish. They can be found by using the residue theory, for example. If any of the imaginary parts of the roots vanish (we have a zero root of multiplicity four for $\Pi_0(\mu)$, for example), then the corresponding functions can be interpreted as the principal values or as the limit cases of roots with positive imaginary parts [8].

Considering the roots $\mu_\nu^{(k)}$ $(k = \pm 1, \pm 2, \ldots)$ simple and $\mathrm{Im}(\mu_\nu^{(k)}) > 0$ $(\nu = 1, 2, 3, 4)$, we find after some manipulation

$$c_0(\alpha) = \frac{1}{2a_9} \left\{\frac{|\alpha|^3}{3!} + \mathrm{Im}\, [(z_1^{(0)})^{-3} e^{iz_1^{(0)}|\alpha|}]\right\} \quad (1.7)$$

$$c_k(\alpha) = \frac{1}{(ik\omega)^7} \sum_{\nu=1}^{4} \frac{e^{ik\omega z_\nu^{(k)}\alpha}}{\Delta_k'(z_\nu^{(k)})}$$

$$c_{-k}(\alpha) = c_k(-\alpha) = \overline{c_k(\alpha)}, \quad \alpha > 0, \quad k = 1, 2, 3, \ldots$$

$$\Delta_k(z) = \sum_{j=0}^{8} a_j z^{8-j} + \frac{a_9}{(k\omega)^4} [z^4 + 2\lambda z^2 + \lambda^2] = \prod_{\nu=1}^{4} (z - z_\nu^{(k)}) (z - \overline{z_\nu^{(k)}})$$

$$\Delta_k'(z) = \frac{d}{dz} \Delta_k(z), \quad z_\nu^{(k)} = \frac{\mu_\nu^{(k)}}{(k\omega)}, \quad z_1^{(0)} = \frac{\sqrt[4]{a_9}}{\sqrt{2}} (1 + i)$$

The fundamental solution E_0 corresponding to the homogeneous operator L_0 yields the principal part of the fundamental solution E. As above, we find

$$E_0(\alpha, \beta) = \frac{1}{T} \sum_{k=-\infty}^{\infty} g_k(\alpha) e^{ik\omega\beta} \quad (1.8)$$

$$g_0(\alpha) = \frac{|\alpha|^7}{2 \cdot 7!}, \quad g_k(\alpha) = \frac{1}{(ik\omega)^7} \sum_{\nu=1}^{4} \frac{e^{ik\omega z_\nu \alpha}}{\Delta'(z_\nu)}$$

$$g_{-k}(\alpha) = g_k(-\alpha) = \overline{g_k(\alpha)}, \quad \alpha > 0, \quad k = 1, 2, 3, \ldots$$

$$\Delta(z) = \sum_{j=0}^{8} a_j z^{8-j} = \prod_{\nu=1}^{4} (z - z_\nu)(z - \bar{z}_\nu)$$

Formulas (1.8) have been obtained under the assumption that the roots $\Delta(z)$ are simple and $\mathrm{Im}(z_\nu) > 0$.

Expressions for the displacements and strains in a shell due to the effect of a periodic system of concentrated forces Q_k applied at the points $\alpha = \alpha_1$, $\beta = \beta_1 + nT$ ($n = 0, \pm 1, \pm 2 \ldots$) along the α, β axes and the internal normal to the shell surface, respectively, are determined by the relationships

$$u_j(\alpha, \beta) = B_{kj}\Psi_k(\alpha - \alpha_1, \beta - \beta_1) \quad \Psi_k = -\frac{Q_k}{fR_2^2} E, \quad k, j = 1, 2, 3 \tag{1.9}$$

$$\varepsilon_1 = \frac{1}{R_2}\left(\frac{\partial u_1}{\partial \alpha} + \lambda u_3\right), \quad \varepsilon_2 = \frac{1}{R_2}\left(\frac{\partial u_2}{\partial \beta} + u_3\right)$$

Here the operator B_{kj} is the cofactor of L_{kj} in the matrix $[L_{ij}]$.

Let us write down the expression for the deflection at the zero point due to the effect of a force Q_3 at this same point. We have from (1.9) by using the property of the roots $z_\nu^{(k)}$ (see Sect. 2)

$$u_3(0, 0) = -\frac{Q_3}{fR_2^2 T}\left\{\frac{b_0}{2\sqrt{2\sqrt{a_9^3}}} + \frac{2i}{\omega^3}\sum_{k=1}^{\infty}\frac{1}{k^3}\left(\sum_{\nu=1}^{4} q_k(z_\nu^{(k)})\right)\right\} \tag{1.10}$$

$$q_k(z) = \left(\sum_{j=0}^{4} b_j z^{4-j}\right) \Big/ \Delta_k'(z), \quad \left|\sum_{\nu=1}^{4} q_k(z_\nu^{(k)})\right| < C, \quad C > 0$$

where b_j are coefficients of the operator

$$B_{33} = \sum_{j=0}^{4} b_j \frac{\partial^4}{\partial \alpha^{4-j} \partial \beta^j}$$

defined above. The series in (1.10) converges absolutely.

Represented in Fig. 1 are the results of calculating the deflection along lines α due to the effect of two diametrically opposite and equal radial forces for a spherical shell from AG-4S fiberglass ($E_1 = 2.1 \cdot 10^4$ MN/m^2, $E_2 = 1.6 \cdot 10^4$ MN/m^2, $G = 4.1 \cdot 10^3$ MN/m^2, $\nu_2 = 0.07$) for different values of the parameter $\tau = 10^{-2}R_2 / h$ (h is the shell thickness).

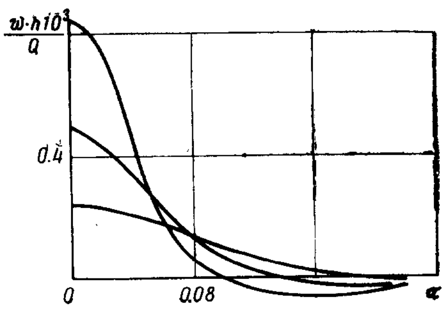

Fig. 1

The expression for the force and strains in the case of loads along the lines is obtained by superposition of appropriate solutions of the concentrated forces. For example

$$\varepsilon_m(\alpha, \beta) = R_2 \int_l \sum_{j=1}^{3} p_j(\alpha_l, \beta_l)\, \varepsilon_m^{(j)}(\alpha - \alpha_l, \beta - \beta_l)\, dl, \; m = 1, 2 \tag{1.11}$$

Here $p_j (j = 1, 2, 3)$ is the intensity of the linear forces distributed over a line l

and directed along the j-th axis, respectively, $\varepsilon_m^{(j)}(\alpha-\alpha_l,\ \beta-\beta_l)$ is the shell strain referred to unit force at a point $(\alpha,\ \beta)$ due to the effect of a system of single concentrated forces Q_j applied at the points $(\alpha_l,\ \beta_l+nT)$.

2. Transformation of the fundamental solution (1.4). Let us establish some of the properties of the roots z_ν and $z_\nu^{(k)}$ of the polynomials $\Delta(z)$ and $\Delta_k(z)$, respectively.

In the case of an anisotropic shell we consider the roots of the polynomial $\Delta(z)$ to be simple and $\mathrm{Im}(z_\nu)>0$ $(\nu=1,\ 2,\ 3,\ 4)$.

Property 1. If z_ν are simple and $\mathrm{Im}(z_\nu)>0$, then starting with some k, at least, the roots of the polynomial $\Delta_k(z)$ are also simple. The estimate ($\varepsilon>0$ is an arbitrarily small number)

$$|z_\nu^{(k)}-z_\nu|<k^{\varepsilon-4},\quad \nu=1,2,3,4 \tag{2.1}$$

holds.

Indeed, by virtue of (1.7) and (1.8), the polynomials $\Delta_k(z)$ and $\Delta(z)$ are represented in some closed neighborhood S of points z_ν as

$$\Delta_k(z)=\Delta(z)+k^{-4}q(z)=\Delta(z)+\varphi_k(z) \tag{2.2}$$

$$\Delta(z)=(z-z_\nu)f(z),\quad \lim_{z\to z_\nu}f(z)=\Delta'(z_\nu)\neq 0$$

Let $m=\min_s|f(z)|$. The neighborhood S can be chosen so that $m>0$. Let us consider a neighborhood S_k of points z_ν of radius $k^{\varepsilon-4}$. Then, by virtue of (2.2), starting with some k

$$|\varphi_k(z)|<\frac{m}{k^{4-\varepsilon}}\leqslant|\Delta(z)|,\quad z\in\partial S_k$$

Hence, by the Rouchet theorem, $\Delta(z)$ and $\Delta_k(z)=\Delta(z)+\varphi_k(z)$ have the identical number of roots in S_k.

Property 2. Under the same assumptions relative to z_ν the identities

$$\sum_{\nu=1}^{4}\left(\frac{z_\nu^{\,q}}{\Delta'(z_\nu)}+\frac{\bar z_\nu^{\,q}}{\Delta'(\bar z_\nu)}\right)=\begin{cases}0, & q=0,1,\ldots,6\\ 1, & q=7\end{cases} \tag{2.3}$$

are valid.

Analogous identities also hold for $z_\nu^{(k)}$. Indeed, let us consider the integral

$$\frac{1}{2\pi i}\oint_L\frac{z^q\,dz}{\Delta(z)}$$

over the closed curve L enclosing all the roots of the polynomial $\Delta(z)$. Evaluating it initially by the Cauchy formula and then by using the residue theorem, we arrive at (2.3).

In order to avoid divergence of the series in the forces and moments, let us proceed as follows. Using (1.7) and (1.8) we represent the fundamental solution $E(\alpha,\ \beta)$ as

$$E(\alpha,\ \beta)=E_0(\alpha,\ \beta)+K(\alpha,\ \beta) \tag{2.4}$$

$$K(\alpha,\beta)=\frac{1}{T}\sum_{k=-\infty}^{\infty}(c_k(\alpha)-g_k(\alpha))\,e^{ik\omega\beta}$$

By using (2.1) and (2.3) it can be proved that the series for $K(\alpha,\ \beta)$ in (2.4) and all its derivatives with respect to $\alpha,\ \beta$ is the seventh order inclusive, converge absolutely and uniformly in any closed neighborhood of zero. The general term of the series attenuates no more slowly than $k^{\varepsilon-3}$ for the seventh order derivatives.

By virtue of (1.8) and (2.3), for example, the relationships

$$\frac{\partial^7 E_0(\alpha,\beta)}{\partial\alpha^{7-j}\partial\beta^j} = \frac{1}{T}\sum_{\nu=1}^{4}\operatorname{Re}\left\{i\,\frac{z_\nu^{7-j}}{\Delta'(z_\nu)}\operatorname{ctg}\frac{\omega}{2}(z_\nu\alpha+\beta)\right\}, \qquad j=1,2,\ldots,7 \quad (2.5)$$

hold for the principal part of the fundamental solution E_0 .

3. Equilibrium of an anisotropic shell reinforced by stiffener ribs. Let us consider a shell closed along β , which is reinforced along the congruent segments $-l/2 \leqslant \alpha_s \leqslant l/2,\ \beta_s = sT\ (s=0, 1, \ldots, \omega-1)$ by thin stiffeners loaded at the ends $\alpha = -l/2$ by identical longitudinal forces Q along the negative α-axis. Let us assume that the rib is connected continuously to the shell and is effective only under tension.

Let us investigate the nature of the force distribution in the shell and in the rib. This problem has been solved in [6] in the case of an isotropic cylindrical shell.

Let $q(\alpha)$ be the longitudinal tangential force transmitted from the rib to the shell. The strain compatibility condition for the rib and the shell on the contact line and the static condition for the stiffener rib are

$$-\int_{\alpha}^{l/2} q(\alpha)\,d\alpha = \frac{E_0F_0}{R_2}\,\varepsilon_1(\alpha,0) \quad (3.1)$$

$$\int_{-l/2}^{l/2} q(\alpha)\,d\alpha = -\frac{Q}{R_2} \quad (3.2)$$

Here E_0, F_0 are the elastic modulus and cross-sectional area of the rib.

Substituting the expression for the strain $\varepsilon_1(\alpha, 0)$ from (1.11) into (3.1), using the representation (2.4), (2.5) and introducing the change of variable $\alpha = lx/2$, we arrive at a singular integral equation

$$\int_{-1}^{1}\frac{\varphi(t)\,dt}{t-x} + A\int_{-1}^{1}\Phi(x-t)\varphi(t)\,dt + B\int_{x}^{1}\varphi(t)\,dt = 0, \qquad \varphi(x) = q(\alpha) \quad (3.3)$$

$$\Phi(x) = \sum_{j=1}^{3}\Phi_j\left(\frac{l}{2}x\right), \qquad A = -\frac{\omega l}{4}\left(i\sum_{\nu=1}^{4}\frac{d(z_\nu)}{z_\nu}\right)^{-1}, \quad B = -\frac{R_2T}{E_0F_0}A$$

$$\Phi_1(x) = \frac{\operatorname{sign} x}{2}\left(A_0 - \frac{B_0}{a_0}\right)\operatorname{Re}\{e^{iz_1^{(0)}|x|} - 1\}$$

$$\Phi_2(x) = \operatorname{Re}\left\{i\sum_{\nu=1}^{4} d(z_\nu)\sum_{m=1}^{\infty}\frac{4\omega z_\nu x}{(\omega z_\nu x)^2-(2\pi m)^2}\right\}$$

$$\Phi_3(x) = 2\operatorname{sign} x\sum_{k=1}^{\infty}\operatorname{Re}\left\{\sum_{\nu=1}^{4}\left[\left(d_k(z_\nu^{(k)}) + \frac{p_k(z_\nu^{(k)})}{(k\omega)^4}\right)e^{ik\omega z_\nu^{(k)}|x|} - d\,(z_\nu)e^{ik\omega z_\nu|x|}\right]\right\}$$

$$d_k(z) = \frac{1}{\Delta_k'(z)}\sum_{j=0}^{6}A_jz^{7-j}, \qquad d(z) = \frac{1}{\Delta'(z)}\sum_{j=0}^{6}A_jz^{7-j}$$

$$p_k(z)=\frac{1}{\Delta_k'(z)}\sum_{j=0}^{3}B_j z^{3-j}$$

Here A_j and B_j are coefficients of the operator

$$(fR_2^2)^{-1}\left(\frac{\partial B_{11}}{\partial\alpha}+\lambda B_{13}\right)=\sum_{j=0}^{6}A_j\frac{\partial^7}{\partial\alpha^{7-j}\partial\beta^j}+\sum_{j=0}^{3}B_j\frac{\partial^3}{\partial\alpha^{3-j}\partial\beta^j}$$

In combination with the additional condition (3.2), Eq. (3.3) determines the stress and strain in the shell and rib uniquely.

The singular integral equation (3.3) can be regularized according to Carleman-Vekua by reduction to a Fredholm equation of the second kind. The arbitrary constant appearing during the regularization is determined by the relationship (3.2).

However, it is apparently more expedient to use one of the procedures for direct solution of singular equations [9—11] for a numerical realization of the algorithm. The procedure developed in [11] ia applied below.

Let us assume

$$\varphi(x)=\varphi_0(x)/\sqrt{1-x^2} \tag{3.4}$$

Here $\varphi_0(x)$ is Hölder-continuous in $[-1, 1]$.

Substituting (3.4) into (3.3) and introducing the new variable ϑ by means of $x=\cos\vartheta$, $0\leqslant\vartheta\leqslant\pi$ we reduce it to

$$\int_0^{\pi}\frac{\varphi_0(\cos\tau)\,d\tau}{\cos\tau-\cos\vartheta}+A\int_0^{\pi}k(\vartheta,\tau)\,\varphi_0(\cos\tau)\,d\tau+B\int_0^{\vartheta}\varphi_0(\cos\tau)\,d\tau=0 \tag{3.5}$$

$$k(\vartheta,\tau)=\Phi(\cos\vartheta-\cos\tau)$$

Having constructed a Lagrange interpolation polynomial for the desired function $\varphi_0(x)$ at the Chebyshev nodes

$$L_n[\varphi_0, x]=\frac{2}{n}\sum_{\nu=1}^{n}\varphi_\nu^0\sum_{m=0}^{n-1}\cos m\vartheta_\nu\cos m\vartheta-\frac{1}{n}\sum_{\nu=1}^{n}\varphi_\nu^0$$

$$\varphi_\nu^0=\varphi_0(x_\nu),\quad x_\nu=\cos\vartheta_\nu,\quad \vartheta_\nu=\frac{2\nu-1}{2n}\pi,\quad \nu=1,2,\ldots,n$$

and using the relationship [11]

$$\frac{1}{\pi}\int_0^{\pi}\frac{\cos n\tau d\tau}{\cos\tau-\cos\vartheta}=\frac{\sin n\vartheta}{\sin\vartheta},\quad 0\leqslant\vartheta\leqslant\pi,\quad n=1,2,\ldots$$

$$\int_{-1}^{1}\frac{F(x)\,dx}{\sqrt{1-x^2}}=\frac{\pi}{n}\sum_{\nu=1}^{n}F(\cos\vartheta_\nu)$$

(the last formula is valid when $F(x)$ is a polynomial of order $\leqslant 2n-1$), we derive the quadrature formulas

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{\varphi(t)\,dt}{t-x}=\frac{1}{n\sin\vartheta}\sum_{\nu=1}^{n}\varphi_\nu^0\sum_{m=0}^{n-1}\cos m\vartheta_\nu\sin m\vartheta \tag{3.6}$$

$$\frac{1}{2\pi}\int_{-1}^{1}\Phi(x-t)\,\varphi(t)\,dt=\frac{1}{2n}\sum_{\nu=1}^{n}k(\vartheta,\vartheta_\nu)\,\varphi_\nu^0$$

$$\frac{1}{2\pi}\int_x^1 \varphi(t)\,dt = \frac{1}{\pi n}\sum_{\nu=1}^{n}\varphi_\nu^0\left[\sum_{m=1}^{n-1}\frac{\cos m\vartheta_\nu \sin m\vartheta}{m} + \frac{\vartheta}{2}\right]$$

The expressions (3.6) afford a possibility of replacing (3.2) and (3.3) by a system of linear algebraic equations in the approximate values of the desired function at the nodal points.

After some manipulation, this system becomes

$$\frac{\pi}{n}\sum_{\nu=1}^{n}\varphi_\nu^0 = -\frac{2Q}{lR_2}, \quad \sum_{\nu=1}^{n}\alpha_{m\nu}\varphi_\nu^0 = 0, \quad m = 1, 2, \ldots, n \tag{3.7}$$

$$\alpha_{m\nu} = \frac{1}{2n}\left[\frac{1}{\sin\vartheta_\nu}\operatorname{ctg}\frac{\vartheta_m + (-1)^{|m-\nu|}\vartheta_\nu}{2} + Ak(\vartheta_m\vartheta_\nu) + \right.$$
$$\left.\frac{2B}{\pi}\left(\sum_{j=1}^{n-1}\frac{1}{j}\cos j\vartheta_\nu \sin j\vartheta_m + \frac{\vartheta_m}{2}\right)\right]$$

4. Results of the computations. The computations were carried out on a M-222 digital computer. We assumed $n = 20$ and $n = 30$ in the system (3.7), which corresponds to partition of the interval into 20 and 30 Chebyshev nodes, respectively. The solutions agree to the accuracy of the fourth symbol.

Represented in Fig. 2 are results of computing the tangential stress distribution q along a rib (Fig. 2a) and the dimensionless force in the stiffener (Fig. 2b)

$$P(\mu) = -\frac{R_2}{Q}\int_\mu^l q\left(\mu - \frac{l}{2}\right)d\mu, \quad 0 < \mu < l$$

for a AG-4S fiberglass cylindrical shell with $l = L / R_2 = 1;\ 2$ (L is the rib length), $\omega = 6;\ 40$ (ω is the quantity of ribs, curve *1* corresponds to $\omega = 40$ for $l = 1$) and the relative stiffness $U = E_1R_2^2 / E_0F_0 = 2\cdot10^2;\ 0$ (the solid lines correspond to the value $U = 2\cdot10^2$, and the dashes to $U = 0$).

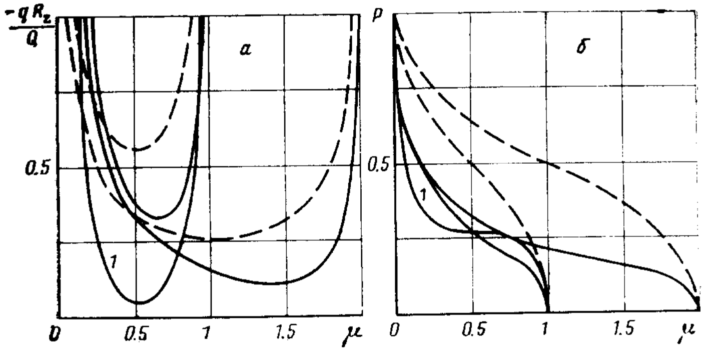

Fig. 2

5. Construction of the Green's function for a finite shell. Let us analyze a shallow anisotropic shell closed in β and finite in α. The solution of the question about the equilibrium of such a shell subjected to a system of equal forces, T-periodic in β, applied at the points $(\alpha_1, \beta_1 + nT)$, $n = 0, 1, \ldots, \omega - 1$ reduces to constructing the fundamental solution E^* of the operator L with given boundary conditions for $\alpha = 0, l$.

For example, we have by virtue of (1.3) for the case of radial forces acting on a shell with moving hinge-supported edges

$$L\left(\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}\right) E^*(\alpha, \beta; \alpha_1, \beta_1) = \delta(\alpha-\alpha_1)\,\delta_T(\beta-\beta_1) \tag{5.1}$$

$$\left(\frac{\partial B_{13}}{\partial\alpha} + \frac{C_{16}}{C_{11}}\left(\frac{\partial B_{23}}{\partial\alpha} + \frac{\partial B_{13}}{\partial\beta}\right)\right) E^* = \left(\frac{\partial^2}{\partial\alpha^2} + 2\frac{D_{16}}{D_{11}}\frac{\partial^2}{\partial\alpha\,\partial\beta}\right) B_{33}E^* =$$

$$B_{23}E^* = B_{33}E^* = 0 \quad \text{при } \alpha = 0, l$$

$$B_{13} = \sum_{j=0}^{3} e_j \frac{\partial^3}{\partial\alpha^{3-j}\,\partial\beta^j}, \quad B_{23} = \sum_{j=0}^{3} d_j \frac{\partial^3}{\partial\alpha^{3-j}\,\partial\beta^j}, \quad B_{33} = \sum_{j=0}^{4} b_j \frac{\partial^4}{\partial\alpha^{4-j}\,\partial\beta^j}$$

Let us consider the construction of the Green's function for this case in detail. Let us seek E^* as

$$E^* = \frac{1}{T}\sum_{k=-\infty}^{\infty} C_k(\alpha, \alpha_1)\, e^{ik\omega(\beta-\beta_1)} \tag{5,2}$$

Substituting (5.2) into (5.1) and equating coefficients of identical powers of $e^{ik\omega(\beta-\beta_1)}$ we obtain the following boundary value problem

$$L_k(C_k) = \sum_{j=0}^{8} a_j (ik\omega)^j \frac{d^{8-j}C_k}{d\alpha^{8-j}} + \tag{5.3}$$

$$a_9\left[\frac{d^4C_k}{d\alpha^4} + 2\lambda(ik\omega)^2\frac{d^2C_k}{d\alpha^2} + \lambda^2(ik\omega)^4C_k\right] = \delta(\alpha-\alpha_1)$$

$$U_j(C_k) = D_j(C_k)|_{\alpha=0} = 0, \quad U_{j+4}(C_k) = D_j(C_k)|_{\alpha=l} = 0, \quad j = 1, 2, 3, 4$$

$$D_1 = \sum_{j=0}^{3} d_j (ik\omega)^j \frac{d^{3-j}}{d\alpha^{3-j}}, \quad D_2 = \sum_{j=0}^{4} b_j (ik\omega)^j \frac{d^{4-j}}{d\alpha^{4-j}},$$

$$D_3 = \left(\frac{d^2}{d\alpha^2} + 2(ik\omega)\frac{D_{16}}{D_{11}}\frac{d}{d\alpha}\right) D_2$$

$$D_4 = \left(\frac{d}{d\alpha} + \frac{C_{16}}{C_{11}}(ik\omega)\right)\sum_{j=0}^{3} e_j (ik\omega)^j \frac{d^{3-j}}{d\alpha^{3-j}} + \frac{C_{16}}{C_{11}}\frac{d}{d\alpha}D_1, \quad k = 0, \pm 1, \pm 2, \ldots$$

As before, considering the roots $Z_\nu^{(k)}$ simple, let us write down the fundamental system of solutions of the operator L_k

$$y_{\nu,k} = e^{ik\omega z_\nu^{(k)}\alpha}, \quad y_{\nu+4,k} = e^{ik\omega \overline{z_\nu^{(k)}}\alpha}, \quad \nu = 1, 2, 3, 4$$

Then the solution of the boundary value problem (5.3) becomes [12]

$$C_k(\alpha, \alpha_1) = \frac{Y_k(\alpha, \alpha_1)}{\Delta_k}, \quad c_k(\alpha, \alpha_1) = c_k(\alpha - \alpha_1)$$

$$Y_k(\alpha, \alpha_1) = \begin{vmatrix} c_k(\alpha, \alpha_1) & y_{1,k}(\alpha) & \ldots & y_{8,k}(\alpha) \\ U_1(c_k) & U_1(y_{1,k}) & \ldots & U_1(y_{8,k}) \\ \ldots & \ldots & \ldots & \ldots \\ U_8(c_k) & U_8(y_{1,k}) & \ldots & U_8(y_{8,k}) \end{vmatrix}, \quad \Delta_k = \begin{vmatrix} U_1(y_{1,k}) & U_1(y_{2,k}) & \ldots & U_1(y_{8,k}) \\ U_2(y_{1,k}) & U_2(y_{2,k}) & \ldots & U_2(y_{8,k}) \\ \ldots & \ldots & \ldots & \ldots \\ U_8(y_{1,k}) & U_8(y_{2,k}) & \ldots & U_8(y_{8,k}) \end{vmatrix}$$

It hence follows that the function $C_k(\alpha, \alpha_1)$ can be represented as the sum of the fundamental solution of the operator L_k constructed in Sect. 1 and the regular solution of equation $L_k(y)$ taking account of the influence of the boundary conditions

$$C_k(\alpha, \alpha_1) = c_k(\alpha, \alpha_1) + \sum_{j=1}^{8} y_{j,k} \frac{\Delta_{j,k}}{\Delta_k}$$

Here $\Delta_{j,k}$ is the cofactor of the element $y_{j,k}$ in the determinant $Y_k(\alpha, \alpha_1)$.
The solution for other boundary conditions is also constructed in an analogous manner.

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